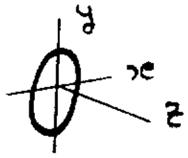
7 Geometric properties, from tables:



$I_{xx} = I_{yy} = \pi R^3$; $J_{zz} = 2\pi R^3$

The rim is symmetric ($I_{xy} = 0$) so (4a) takes the form :-

$$\mathbf{b} = -[M_x/I_{xx} \;\; M_y/I_{yy} \;\; M_z/J_{zz}]' = -\frac{1}{\pi R^3}[M_x \;\; M_y \;\; \tfrac{1}{2}M_z]'$$

For the point P, $\mathbf{r} = R[\cos\phi \;\; \sin\phi \;\; 0]'$

So, from (2) and (3) the intensity at P is :-

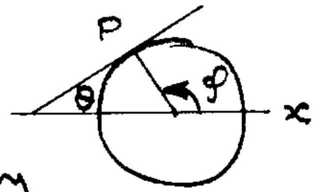
$$\mathbf{q} = \mathbf{b}\times\mathbf{r} - \mathbf{F}/L \quad ; \quad L = 2\pi R$$

$$= -\frac{1}{\pi R^3}[M_x \;\; M_y \;\; \tfrac{1}{2}M_z]' \times R[\cos\phi \;\; \sin\phi \;\; 0]' - \frac{1}{L}[F_x \;\; F_y \;\; F_z]'$$

or, introducing the notation $M = Rm$

$$= -\frac{1}{L}\left([2m_x \;\; 2m_y \;\; m_z]' \times [\cos\phi \;\; \sin\phi \;\; 0]' + [F_x \;\; F_y \;\; F_z]'\right)$$

$$= \frac{1}{L}\begin{bmatrix} m_z\sin\phi - F_x \\ -m_z\cos\phi - F_y \\ 2(m_y\cos\phi - m_x\sin\phi) - F_z \end{bmatrix} \quad \text{i.e.} \; \equiv \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix}$$



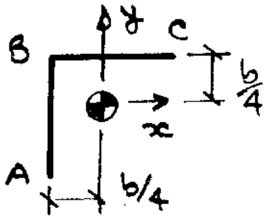
This must be resolved into local components via the $\mathbf{t}$-matrix (eq 5), which incorporates the slope θ. Evidently $\theta = \phi - \pi/2$ and so, from (5a) $\mathbf{t} = \begin{bmatrix} \sin\phi & -\cos\phi & 0 \\ \cos\phi & \sin\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

So, employing (5) to find local components:

$$\mathbf{q} \equiv \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \mathbf{t}\mathbf{q} = \frac{1}{L}\begin{bmatrix} \sin\phi & -\cos\phi & 0 \\ \cos\phi & \sin\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} m_z\sin\phi - F_x \\ -m_z\cos\phi - F_y \\ 2(m_y\cos\phi - m_x\sin\phi) - F_z \end{bmatrix}$$

which, when multiplied out, gives the quoted solution.

8 Geometric properties, from tables ($d = b$).



Centroid located as shown. $L = 2b$.

$I_{xx} = I_{yy} = \frac{5}{24}b^3$; $I_{xy} = \frac{1}{8}b^3$

Centroidal loading:

$\mathbf{F} = [0 \;\; 0 \;\; F]'$; $\mathbf{M} = [\tfrac{1}{4}bF \;\; \tfrac{1}{4}bF \;\; 0]'$

$\mathbf{b}$-vector, using (4a) : $I_0^2 = ((5/24)^2 - (1/8)^2)b^6 = b^6/36$

$$\mathbf{b} = -\left[(\tfrac{1}{4}bF\cdot\tfrac{5}{24}b^3 + \tfrac{1}{4}bF\cdot\tfrac{1}{8}b^3)/\tfrac{1}{36}b^6 \;\; \text{ditto} \;\; 0\right]' = -\frac{3F}{b^2}[1 \;\; 1 \;\; 0]'$$

By symmetry the intensities in AB will be the same as in BC. Consider the latter as $\mathbf{t}$ simpler.

<u>Line B-C</u> $\theta = 0$, so $\mathbf{t} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Point B, $\mathbf{r} = \frac{b}{4}[-1 \;\; 1 \;\; 0]'$

From (2) and (3)

$$\mathbf{q} = -\frac{3F}{b^2}[1 \;\; 1 \;\; 0]' \times \frac{b}{4}[-1 \;\; 1 \;\; 0]' - \frac{F}{2b}[0 \;\; 0 \;\; 1]' = \frac{-2F}{b}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \mathbf{t}\mathbf{q} = [0 \;\; 0 \;\; 2F/b]' \quad \therefore q_E = 2F/b.$$

Point C similarly: $\mathbf{r} = b/4[3 \;\; 1 \;\; 0]'$

$$\mathbf{q} = [0 \;\; 0 \;\; F/b]' = \mathbf{t}\mathbf{q} = [0 \;\; 0 \;\; F/b]' \quad \therefore q_E = F/b$$

$\therefore \hat{q}_E = (q_E)_B = 2F/b$ — Insert into (1) with $\sigma_E = S/n$

$$wbS = 4nF \qquad \text{QED}$$