5 cont'd

$$J\dot{\omega} + k(\omega - \bar{\omega}) = -\tilde{T}\,.\sin 2\pi t/t_o$$

Setting $x = 2\pi t/t_o$ and defining the constant $m$ as $m = 2\pi J/k t_o$, the solution to this DE is:

$$(\omega - \bar{\omega})\,k/\tilde{T} = C\exp(-x/m) - \frac{1}{\sqrt{1+m^2}}\sin(x - \tan^{-1} m).$$

Considering only the steady-state component of the RHS, the amplitude of the speed excursion about $\bar{\omega}$ is thus

$$|\omega - \bar{\omega}|_{max} = \frac{\tilde{T}}{k}\cdot\frac{1}{\sqrt{1+m^2}}$$

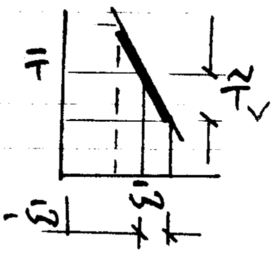
and the corresponding torque amplitude on the motor line is

$$\tilde{T}_V = k\,|\omega - \bar{\omega}|_{max} = \tilde{T}\,\frac{1}{\sqrt{1+m^2}}.$$

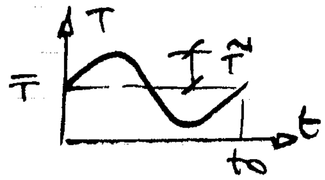
Hence

$$\tilde{T}_V/\tilde{T} = \left[1 + (2\pi J/k t_o)^2\right]^{-1/2}$$

QED.




6 First ascertain the effective torque for a sinusoid of amplitude $\tilde{T}$ and mean $\bar{T}$:




$$T_e^n\,.\,t_o = \int_0^{t_o}\left(\bar{T} + \tilde{T}\sin\frac{2\pi t}{t_o}\right)^n dt$$

with $n = 2$

$$T_e^2\,t_o = \int_0^{t_o}\left(\bar{T}^2 + 2\bar{T}\tilde{T}\sin\frac{2\pi t}{t_o} + \tfrac{1}{2}\tilde{T}^2\left(1 - \cos\frac{4\pi t}{t_o}\right)\right)dt$$

$$= (\bar{T}^2 + \tfrac{1}{2}\tilde{T}^2)\,t_o.$$

i.e. $T_e = \sqrt{(\bar{T}^2 + \tfrac{1}{2}\tilde{T}^2)}$. or use (A2)

If the 180 M motor, with $T_f = 120$ Nm were to drive the punch, with no inertia, $\bar{T} = 110$, $\tilde{T} = 100$ Nm, &

$$T_e = \sqrt{(110^2 + \tfrac{1}{2}\times 100^2)} = 131 \text{ Nm}.$$

– clearly not feasible.

With a view to applying the results of the previous problem, estimate the slope $k$ at the operating point.

The torque-speed equation is:

(a) $$T = T_b/(1 + (s - s_b)^2(a/s - b s^2)) \;;\quad s = 1 - \frac{n}{n_s} = 1 - \frac{\omega}{\omega_s}$$

Differentiating this to get the slope:

$$dT/d\omega = (ds/d\omega)(dT/ds) \;;\quad ds/d\omega = -1/\omega_s$$

$$= -(T^2/\omega_s T_b)(s_b - s)\left[a(s_b + s)/s^2 + 2bs(s_b - 2s)\right]$$

– the second term in the [ ] is negligible, so

$$k = -dT/d\omega = \frac{aT^2}{\omega_s T_b}\left(\left(\frac{s_b}{s}\right)^2 - 1\right)$$

At the mean operating point, $\bar{T}$ of 110 Nm, the corresponding speed is found to be 1473 rpm, from (a) which corresponds to a slip of 0.018. So at this point:

$$k = 1.500\times 110^2\left[\left(\frac{0.179}{0.018}\right)^2 - 1\right]/50\pi\times 348$$

$$= 32.5 \text{ Nms (/rad)}.$$