13 For case (d) $K_I = (6M/bw^2)\, Y\sqrt{\pi a}$

where $Y = (1.12 + \alpha(2.62\alpha - 1.59))/(1-0.7\alpha)$

Note that $6M/bw^2$ is the max. bending stress for a rectangular X-section. Adapting this for a circular X-section, $\sigma = My/I = M\frac{d}{2}/\frac{\pi}{64}d^4$

i.e. $\sigma\,(= \Delta\sigma) = 32\times 200\times 10^3/\pi\times 50^3 = 16.3$ MPa

100 hours corresponds to $100\times 60\times 3000 = 18$ Mc

From (53) applied to basic Paris equation

$$\frac{\Delta N_{12}}{w}\left(\frac{da}{dN}\right)_0\left(\frac{\Delta\sigma\sqrt{\pi w}}{\Delta K_0}\right)^n = \int_1^2 (Y\sqrt{\alpha})^{-n}\, d\alpha$$

$$\therefore \frac{18}{50}\ \frac{\text{Mc}}{\text{mm}}\quad 1\ \frac{\text{mm}}{\text{Mc}}\left(\frac{16.3\sqrt{\pi\times 0.05}}{1.6}\right)^{2.7} = \int_{0.002}^{\alpha_2}(Y\sqrt{\alpha})^{-2.7}\, d\alpha$$

$$\therefore \int_{0.002}^{\alpha_2}(Y\sqrt{\alpha})^{-2.7}\, d\alpha = 15.6$$

Evaluating the integral numerically, with trial upper bounds, α_2, until the equation is satisfied gives $\alpha_2 = 0.225$ $\therefore$ $a = 0.225\times 50 = \underline{11.3}$ mm

14 Adapting (53) as in notes, with the integral:-

$I \equiv \int_1^2 (Y\sqrt{\alpha})^{-n}\, d\alpha = \int_1^2\left(\frac{1-\alpha}{Y_0}\frac{1}{\sqrt{\alpha}}\right)^4 d\alpha$; $Y_0 = 0.84$

$$= \frac{1}{Y_0^4}\int_1^2 \frac{1-4\alpha+6\alpha^2-4\alpha^3+\alpha^4}{\alpha^2}\, d\alpha = \frac{1}{Y_0^4}\int_1^2\left[\frac{1}{\alpha^2}-\frac{4}{\alpha}+6-4\alpha+\alpha^2\right]d\alpha$$

$$= \frac{1}{Y_0^4}\left[-\frac{1}{\alpha} - 4\ln\alpha + 6\alpha - 2\alpha^2 + \frac{1}{3}\alpha^3\right]_1^2$$

with limits $\alpha_1 = 5/20 = 0.25$; $\alpha_2 = 15/20 = 0.75$

$$I = \frac{1}{0.84^4}\left[\frac{1}{0.25}-\frac{1}{0.75} - 4\ln 3 + 6.\tfrac{1}{2} - 2\left(\tfrac{9}{16}-\tfrac{1}{16}\right) + \tfrac{1}{3}\left(\tfrac{27}{64}-\tfrac{1}{64}\right)\right] = 0.819$$

Inserting this into (53) with last term irrelevant:-

$\frac{\Delta N_{12}}{20}\times 1\left(\frac{40\sqrt{0.02\pi}}{6}\right)^4 = 0.819$ whence $\Delta N_{12} = \underline{2.10}$ Mc

Similarly (b) if $\Delta\sigma$ is halved from 40 to 20 MPa then $\Delta N_{12} = 2.10\times 2^4 = \underline{33.6}$ Mc

15 Now take instability into account (elastic instability only) The critical crack length, α_c is:

$K_{Ic} = \hat{\sigma}\, Y\sqrt{\pi a}\,\big|_c$ — solve for α_c

a) $60 = 40\,\frac{0.84}{1-\alpha_c}\sqrt{\pi\times 0.02\,\alpha_c}$

Solving by trial-and-error gives $\alpha_c = 0.869$

Insert into (53) with limit '2' changed to 'c'

From the previous problem, the RHS is

$$0.84^{-4}\left[-1/\alpha - 4\ln\alpha + 6\alpha - 2\alpha^2 + \alpha^3/3\right]_{0.25}^{0.869}$$

$$-\left[\frac{0.84}{1-0.869}\sqrt{0.869}\right]^{-4}(0.869 - 0.25)$$

$= 0.819$

$\therefore (\Delta N_{12}/20)\left[\frac{40\sqrt{\pi\times 0.02}}{6}\right]^4 = 0.819$ whence $\Delta N_{12} = 2.10$ Mc.