8 (cont'd) a little manipulation :-

$m + n(n+2\alpha) = (1-\alpha)^2$ RQD.

where m & n are as defined in problem statement. The significance of this equation is that it gives, for a certain crack size, the combinations of tension & bending which will result in plastic collapse. Note that even with M=0, bending will be set up in the ligament. Note too that either or both M and N may be ±. For π_1, π_2 both $\leq w-a$, from above $|m| \leq 1-\alpha$ and inversely from final equation, $n = \hat{n} = (1-\alpha)^2$ when $n=0$, ∞ $|m| \leq (1-\alpha)^2$

9. Find the failure loads under separate mechanisms of elastic instability & plastic collapse, then use the R6 interaction. However elastic effects are not expected to be very significant since $b_0 = 2.5(K_{Ic}/\sigma_y)^2 = 8$mm which is smaller than salient dimensions, i.e. plane strain prevails. Work in stresses rather than P, since erratic.

ELASTIC $K_{Ic} = \sigma_E Y\sqrt{\pi a}$; Y from eqn (1)

i.e. $\sigma_E = 40/Y\sqrt{\pi \times 0.025\alpha} = 142.7/Y\sqrt{\alpha}$ MPa

PLASTIC From result of Q8 :-

$\sigma_P = \sigma_y(1-\alpha)^2 = 700(1-\alpha)^2$ MPa

If a = 2mm then $\alpha = a/w = 2/25 = 0.08$

$\therefore Y = 1.142$; $\sigma_E = 442$ MPa & $\sigma_P = 592$ MPa

Insert in R6 equation (4) $\Rightarrow \sigma_F = 392$ MPa

$\therefore P_F = \sigma_F A = 3.92 \times \frac{\pi}{4} \times 50^2 = \underline{773}$ kN

The failure stress corr. to 2500kN = 102 MPa.

Using trial & error approach :-

trial α	0.1	0.5	0.45	0.49	0.495
σ_E	392	108	126	112	110
σ_P	567	175	212	182	179
σ_F from (4)	355	100	117	103	102 ✓

$\therefore$ Critical crack size is $0.495 \times 25 = \underline{12.4}$ mm

10. Work out the failure load, M_F, via R6, then compare (a), (c) with (b).

ELASTIC. eqn (a) $K_{Ic} = (6M_E/bw^2)Y\sqrt{\pi a}$

where $Y = 1.054$ $\therefore M_E/bw^2 = K_{Ic}/6 \times 1.054\sqrt{\pi \times 0.002}$

i.e. $M_E/bw^2 = 1.994\, K_{Ic}$

PLASTIC for plastic collapse from results of problem 8 with n=0 :- $m_P = 2M_P/bw^2\sigma_y = \pm(1-\alpha)^2$

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