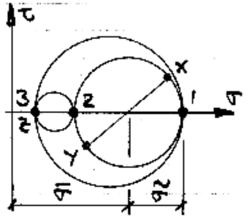
4 We shall make the substitutions:

$\phi = \theta/2$; $\sigma_0 = (K_0 \cos\theta/2)/\sqrt{2\pi r}$

so that equations (1) take the form:

$$\begin{cases}\sigma_x = \sigma_0 (1 - \sin\phi \sin 3\phi) \\ \sigma_y = \sigma_0 (1 + \sin\phi \sin 3\phi) \\ \tau_{xy} = \sigma_0 (\sin\phi \cos 3\phi)\end{cases}$$



Resolving stresses in x-y plane

$\bar{\sigma} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_0$

$\tilde{\sigma} = \sqrt{\left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]} = \sigma_0 \sin\phi$

and so the principals are:

$\sigma_1 = \bar{\sigma} + \tilde{\sigma} = \sigma_0(1 + \sin\phi)$

$\sigma_2 = \bar{\sigma} - \tilde{\sigma} = \sigma_0(1 - \sin\phi)$

– giving rise to the Mohr's circles shown, & in which we recognise either

plane stress : $\sigma_z = 0$

or plane strain : $\varepsilon_z = 0$ i.e. $\sigma_z = \nu(\sigma_x + \sigma_y) = 2\nu\sigma_0$

That is, both cases are covered by:

$\sigma_3 = \sigma_z = k\sigma_0$; $k = 0$ (plane stress) or $= 2\nu$ (plane strain)

Inserting the three principals into the distortion energy failure theory :–

$$2\sigma_E^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_0^2[(1-k)^2 + 3\sin^2\phi]$$

Setting $\sigma_E = S_y$ defines the yield boundary. So, substituting from the definition of σ_0

$$S_y^2 = (K_0^2 \cos^2\phi / 2\pi r)\ [(1-k)^2 + 3\sin^2\phi]$$

i.e. the r–θ yield locus is, on rearranging:–

$$r = \frac{1}{2\pi}(K_0/S_y)^2\ [(1-k)^2 + 3\sin^2\phi]\cos^2\phi.$$

which is the desired relation

Note that, although the derivation was based on the infinite plate (K_0), we are justified in generalising it to K_I, since near-tip fields are identical.

5 Reckoning dimensions from load line of action:

$w = 30 - 10 = 20$ mm , $a = 30 - 10 - 9 = 11$ mm $\alpha = 11/20 = 0.55$

& $b = w/2 = 10$ mm (standard proportions)

From case (c), at failure

$$K_c = (P/bw)\, Y\sqrt{\pi a} = \frac{10^4}{10 \times 20} \times \frac{5.23 + 0.55(5.10 \times 0.55 - 5.88)}{1 - 1.07 + 0.55}\sqrt{\pi \times 0.011}$$

$$= 80\ \text{MPa}\sqrt{\text{m}}$$

If this is critical K_{IC} then, from (2a):

$b_0 = 2.5(K_{IC}/S_y)^2 = 2.5(80/1200)^2 \times 10^3 = 11$ mm

Both the thickness & ligament are less than this, so plane stress conditions apply (–in part). The result is valid, but it's NOT a material property.