1. Large sheet, so no geometric effects & $Y = 1$

$K \to K_{Ic}$ when $\sigma = 500$ MPa, $a = 25$ mm

$\therefore K_{Ic} = \sigma\sqrt{\pi a} = 500\sqrt{\pi \times 0.025} = 140$ MPa$\sqrt{m}$

Having established this critical value, then

$\sigma_c = K_{Ic}/\sqrt{\pi a} = 140/\sqrt{\pi \times 0.05} = \underline{354}$ MPa.

2 Using a safety factor of $n = 1.5$ implies that the casings will be put into service with a background stress of $\sigma = S_y/n$.

Assume the configuration is essentially a large plate with $Y = 1$.

Criticality will occur when $K \to K_{Ic}$ assuming LEFM only. Then

$K_{Ic} = \sigma\sqrt{\pi a_c}$ or $a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma}\right)^2 = \frac{1}{\pi}\left(\frac{n K_{Ic}}{S_y}\right)^2$

Evaluating this for the two materials:

a) $2a_c = \frac{2}{\pi}(1.5 \times 70/1200)^2 \times 10^3 = 4.9$ mm

b) $2a_c = \frac{2}{\pi}(1.5 \times 50/1800)^2 \times 10^3 = 1.1$ mm

Since the design stress is a constant fraction of the yield, then material toughnesses must increase in proportion to yields, if materials are to possess the same defect tolerance. However an inverse relationship usually applies, as it does here. That is, a high strength (yield) material is NOT the be-all and end-all when brittle fracture is possible.

3. This is similar to example of Notes – superposition of tension & bending – except that crack size must be found by trial-and-error.

Let $\alpha = a/w$; $w = 100$ mm $= 0.1$ m.

Tension, case (b) — $\sigma = P/A = 250 \times 10^3/100 \times 20 = 125$ MPa

$\therefore K_{It} = \sigma\sqrt{\pi a}\, Y_t = 125\sqrt{\pi \times 0.1}\, Y_t$

where $Y_t = (1.12 + \alpha(2.91\alpha - 0.64))/(1 - 0.93\alpha)$

Bending, case (d) — $\sigma = 6M/bw^2 = 6 \times 250 \times 10^4/20 \times 100^2 = 75$ MPa

$\therefore K_{Ib} = \sigma\sqrt{\pi a}\, Y_b = 75\sqrt{\pi \times 0.1}\, Y_b$

where $Y_b = (1.12 + \alpha(2.62\alpha - 1.59))/(1 - 0.7\alpha)$

So, tabulating

trial α	0.1	0.2	0.15	0.14	0.143	$= \alpha_c$
K_{It}	26.5	42.7	34.4	32.8	33.2	since
K_{Ib}	14.1	19.8	17.1	16.5	16.7	
total	40.6	62.5	51.5	49.3	50.0	$= K_{Ic}$

The critical crack length is $0.143 \times 100 = \underline{14 \text{ mm}}$