14 Adopt shorthand notation $\gamma_i \equiv (\gamma_o)_{inner}$; $\gamma_o = (\gamma_o)_{outer}$.
Optimality will be defined by the maximum equivalent stress under load, $\sigma^* = S_y/n$, being the same for both cylinders.
For both cylinders, since $\bar{\sigma} < \tilde{\sigma}$ then $\sigma_r \le \sigma_z = 0 \le \sigma_t$ i.e. $\sigma^* = 2\tilde{\sigma}$
INNER (p_i and p_c) $\tilde{\sigma} = (p_i - p_c)\gamma_i/(\gamma_i - 1)$ ∴ $\sigma^*_{inner} = 2(p_i - p_c)\gamma_i/(\gamma_i - 1)$
OUTER (p_c and 0) $\tilde{\sigma} = p_c\gamma_o/(\gamma_o - 1)$ ∴ $\sigma^*_{outer} = 2p_c\gamma_o/(\gamma_o - 1)$
Since $\sigma^*_{inner} = \sigma^*_{outer}$ then $(p_i - p_c)\gamma_i/(\gamma_i - 1) = p_c\gamma_o/(\gamma_o - 1)$ – or
(I) $p_c/p_i = \gamma_i(\gamma_o - 1)/(2\gamma_i\gamma_o - \gamma_i - \gamma_o)$ – in which case, from above
(II) $\sigma^*/p_i = 2\gamma_i\gamma_o/(2\gamma_i\gamma_o - \gamma_i - \gamma_o)$.
For an overall diameter ratio $\Gamma = \gamma_i\gamma_o$ we seek to minimise σ^* for a given p_i by suitable choice of γ_i (say). i.e.

$$d(\sigma^*/p_i)/d\gamma_i = \frac{d}{d\gamma_i}\left\{2\Gamma/(2\Gamma - \gamma_i - \Gamma/\gamma_i)\right\} = 0$$

from which $\gamma_i = \sqrt{\Gamma}$, so too, $\gamma_o = \Gamma/\gamma_i = \sqrt{\Gamma}$
Inserting this into (II) and solving for $\sqrt{\Gamma}$ yields

$\sqrt{\Gamma} = 1/(1 - p_i/\sigma^*) = 1/(1 - np_i/S_y)$ – QED.

(b) and into (I) gives $p_c/p_i = 1/2$ at optimum.
This gives optimum contact pressure for given p_i. To find corresponding optimum interference, Δ_{opt}, substitute into (4) [same materials, see equation, example 11]

$$E\Delta/D_c = \left[\frac{\gamma_i+1}{\gamma_i-1} + \frac{\gamma_o+1}{\gamma_o-1}\right]p_c - \frac{2p_i}{\gamma_i-1} \; ; \; \gamma_i = \gamma_o = \sqrt{\Gamma},\ p_c = \tfrac{1}{2}p_i$$

$= p_i$ – i.e. $\Delta_{opt}/D_c = p_i/E$ – QED.

Note that example 11 is close to optimum.