10 cont'd

(c) from (ix) Assume $0 \le p_o/p_i = 0.818 \le 1/\rho_o$ i.e. $\rho_o \le 1.222$

$\therefore (\rho_o - 1) S_{uc}/n = [(\rho_o+1)p_i - 2\rho_o p_o] m$ $\therefore (\rho_o - 1)\frac{700}{4} = \frac{7}{2}[(\rho_o+1)55 - 2\rho_o \times 45]$

which yields $\rho_o = 21/17 = 1.235$ which is not ≤ 1.222

so Assume $1/\rho_o \le p_o/p_i = 0.818 \le (1+1/\rho_o)/2$ i.e. $1.222 \le \rho_o \le 1.571$

$\therefore (\rho_o - 1) S_{uc}/n = [(\rho_o+1)m - 2]p_i - 2(m-1)\rho_o p_o$

$175(\rho_o - 1) = [\frac{7}{2}(\rho_o+1) - 2]55 - 5\rho_o 45 \Rightarrow \rho_o = 1.241$ (OK) $D_o = 56$ mm

11. When the materials of both components of a composite cylinder are identical, equation (4) degenerates to:

$$\left[\frac{\rho_o+1}{\rho_o-1}\Big|_{inner} + \frac{\rho_o+1}{\rho_o-1}\Big|_{outer}\right] p_c = \frac{\Delta E}{D_c} + 2\left[\frac{p_i}{\rho_o-1}\Big|_{inner} + \frac{\rho_o p_o}{\rho_o-1}\Big|_{outer}\right]$$

For the case in hand: ρ_o inner $= 3.24$ ρ_o outer $= 2.25$

so with pressures in MPa:

$4.493\, p_c = 0.05 \times 207 \times 10^3/180 + 2p_i/2.24$

$p_c = 12.8 + 0.1987\, p_i$ MPa.

Hence

	INNER inside	INNER common	OUTER common	OUTER outside
Initial assembly ($p_i = 0$)				
pressure	0	12.8		0
$\hat{\sigma}, \check{\sigma}$ from (2)	−18.5, −18.5		10.2, −23.0	
σ_t from (3)	−37.0	−24.2	33.3	20.5
σ_r from (3)	0	−12.8	−12.8	0
$\sigma^* = \hat{\sigma} - \check{\sigma}$ ($\sigma_z = 0$)	37	24	46	20
Loaded ($p_i = 50$, $p_c = 22.7$ MPa from above)				
pressure	50	22.7		0
$\hat{\sigma}, \check{\sigma}$ from (2)	−10.6, 39.4		18.2, 40.9	
σ_t from (3)	28.9	1.6	59.1	36.4
σ_r from (3)	−50	−22.7	−22.7	0
$\sigma^* = \hat{\sigma} - \check{\sigma}$ ($\sigma_z = 0$)	78.9	24.3	<u>81.8</u>	36.4

The stress plot for the composite cylinder in the Notes corresponds to this example. $\sigma^* = 82$ MPa

(a) For a single cylinder of same overall size, $\rho_o = 7.29$

$p_o/p_i = 0$, so from (viii) :– $(\rho_o - 1)\sigma^* = 2\rho_o(p_i - p_o)$

$\therefore p_i = (7.29 - 1) \times 82 / 2 \times 7.29 = \underline{35}$ MPa

(b). As (a) but solve for ρ_o. :– $(\rho_o - 1) \times 82 = 2\rho_o \times 50$

This gives an impossible ρ_o (< 0) so impossible to achieve – no matter how big.

Eg. if $\rho_o \to \infty$ $\sigma^* \Rightarrow 2p_i \Rightarrow 100$ MPa > 82 MPa