9 contd d) ρ_o unknown ; closed, $\sigma_a = \bar{\sigma}$ – a design problem.
Knowns are σ^*, ϵ_{Di}, p_o. Consider σ^* first
ordering principals at the bore :–
$$\sigma_r\ (= \bar{\sigma} - \tilde{\sigma}) \le \sigma_a\ (= \bar{\sigma}) \le \sigma_t\ (= \bar{\sigma} + \tilde{\sigma})$$
Hence $\sigma^* = \hat{\sigma} - \check{\sigma} = 2\tilde{\sigma} = 200$ MPa $\therefore \tilde{\sigma} = 100$ MPa.
From (A) with $\epsilon_{Di} = 6.76 \times 10^{-4}$:
$$6.76 = (7\bar{\sigma} - 3\bar{\sigma} + 13\tilde{\sigma})/207$$
$$\therefore 4\bar{\sigma} = 1400 - 13 \times 100 \qquad \therefore \bar{\sigma} = 25 \text{ MPa}.$$
Finally $-p_o = 0 = \bar{\sigma} - \tilde{\sigma}/\rho_o \qquad \therefore \rho_o = 4.$
Hence other stresses and strains as above.

e). ρ_o unknown – a design problem.
Closed, so $\sigma_a = \bar{\sigma} - \sigma_F$ (assuming hydrostatic, see (i))
where $\sigma_F = F/A_a = F_a/\frac{\pi}{4}(D_o^2 - D_i^2)$ – D_o given
$$= \frac{4F_a}{\pi D_o^2}\frac{1}{1 - 1/\rho_o} = \frac{4 \times 600 \times 10^3}{\pi \times 120^2}\frac{\rho_o}{\rho_o - 1} = \frac{53.05\rho_o}{\rho_o - 1} \text{ MPa}$$
Also given $p_i = 0 = \sigma_{ri} = \bar{\sigma} - \tilde{\sigma}$ i.e. $\bar{\sigma} = \tilde{\sigma}$
given $\epsilon_{Do} = 0$, so using (A)
$0 = 7\bar{\sigma} - 3(\bar{\sigma} + \sigma_F) + 13\tilde{\sigma}/\rho_o$ and $\bar{\sigma} = \tilde{\sigma}$ from above.
$\therefore \tilde{\sigma}(4 + 13/\rho_o) = 3\sigma_F$ and σ_F from above gives
$\bar{\sigma} = \tilde{\sigma} = -159.2\rho_o^2/(\rho_o - 1)(4\rho_o + 13)$ MPa.
We now have expressions for $\bar{\sigma}$, $\tilde{\sigma}$ & σ_F in ρ_o terms.
So look at max. equivalent stress at bore, $\sigma^* = 100$ MPa
$$\sigma_t = \bar{\sigma} + \tilde{\sigma} = 2\tilde{\sigma} = -318.4\rho_o^2/(\rho_o - 1)(4\rho_o + 13).$$
$$\sigma_r = 0 \quad \text{since } p_i = 0$$
$\sigma_a = \bar{\sigma} - \sigma_F = -53.05\rho_o(7\rho_o + 13)/(\rho_o - 1)(4\rho_o + 13)$ from above.
Evidently $\hat{\sigma} = \sigma_r = 0$ so $\sigma^* = |\check{\sigma}|$ but it is
not clear which is $\check{\sigma}$ — σ_t or σ_a – so try each.
If $\check{\sigma} = \sigma_t$ $\quad 318.4\rho_o^2/(\rho_o - 1)(4\rho_o + 13) = 100 \Rightarrow \rho_o = 1.293$
in which case $\sigma_a = -284$ MPa
– i.e. assumption is invalid.
If $\check{\sigma} = \sigma_a$ $\quad 53.05\rho_o(7\rho_o + 13)/(\rho_o - 1)(4\rho_o + 13) = 100 \Rightarrow \rho_o = 4$
in which case $\sigma_t = -59$ MPa – OK.
So $\rho_o = 4$, $\bar{\sigma}$, $\tilde{\sigma}$, & σ_F follow from above, hence completion

10. a design problem $p_i = 55$ MPa, $p_o = 45$ MPa – open.
(a) from (vii) $\sigma^* = S_y/n = 200/4 = 50$ MPa.
$$\sigma^* = 50 = \sqrt{[(p_i - \rho_o p_o)^2 + 3\rho_o^2(p_i - p_o)^2]}/(\rho_o - 1) = 5\sqrt{[(11 - 9\rho_o)^2 + 12\rho_o^2]}/(\rho_o - 1)$$
which yields $\rho_o = (1 + \sqrt{148})/7 = 1.881 \quad \therefore D_o = \sqrt{\rho_o}\,D_i = 69$ mm
(b) from (viii) Assume $0 \le p_o/p_i = 0.818 \le (1 + 1/\rho_o)/2$ i.e. $\rho_o \le 1.571$
$$(\rho_o - 1)\sigma^* = 2\rho_o(p_i - p_o). \quad ; \quad 50(\rho_o - 1) = 20\rho_o$$
which yields $\rho_o = 1.67$ which is not ≤ 1.571
so Assume $(1 + 1/\rho_o)/2 \le p_o/p_i = 0.818 \le 1$ i.e. $1.571 \le \rho_o$.
$$\sigma^* = p_i = 55 \text{ MPa} > 50 \text{ MPa}.$$
i.e. no cylinder possible with a safety factor of 4.