9. In the following, we seek to establish at the earliest opportunity values for γ_0, $\bar{\sigma}$ and $\tilde{\sigma}$. Strains then follow from (x)

(A) $10^4 \epsilon_\theta = (7\bar{\sigma} - 3\sigma_\theta + 13\tilde{\sigma}/\gamma)/207$

so does the maximum equivalent stress at the bore

(B) $\sigma^* = \hat{\sigma} - \check{\sigma}$

where $\hat{\sigma} = \max(\sigma_t, \sigma_\theta, \sigma_r)|_{\gamma=1}$, $\check{\sigma} = \min(\sigma_t, \sigma_\theta, \sigma_r)|_{\gamma=1}$

a) $\gamma_0 = (120/40)^2 = 9$; open so $\sigma_\theta = 0$

from (2) $\bar{\sigma} = -30$ $\tilde{\sigma} = -90$ MPa.

		at bore ($\gamma=1$)	at OD ($\gamma=9$)	
from (3)	$\sigma_t = \bar{\sigma} + \tilde{\sigma}/\gamma$	60 ✓	−20 ✓	MPa
	$\sigma_r = \bar{\sigma} - \tilde{\sigma}/\gamma$	−120 (= $-p_i$)	−40 (= $-p_0$)	MPa
from (A)	$10^4\epsilon_\theta = (7\bar{\sigma} + 13\tilde{\sigma}/\gamma)/207$	4.6	−0.4	

from (B) $\check{\sigma} = -120$ $\hat{\sigma} = 60$ $\therefore \sigma^* = 60 - (-120) = 180$ MPa.

b) $\gamma_0 = (60/20)^2 = 9$; open $\therefore \sigma_\theta = 0$

Evaluate $\bar{\sigma}$, $\tilde{\sigma}$ from p_i, $\epsilon_{\theta 0}$ given

from (3) at $\gamma = 1$ $\bar{\sigma} - \tilde{\sigma} = -p_i = -100$

from (A) at γ_0 $7\bar{\sigma} + 13\tilde{\sigma}/9 = 0$

$\Rightarrow \bar{\sigma} = -32.5$ MPa, $\tilde{\sigma} = 157.5$ MPa

Hence stresses and strains as in (a) above, noting that $p_0 = -\sigma_r|_{\gamma_0} = 50$ MPa

At bore $\check{\sigma} = -100$ $\hat{\sigma} = 125$ $\therefore \sigma^* = 315$ MPa

c) γ_0 not known, but $\sigma^* = 160$ MPa – a design problem. One could proceed as in previous examples, retaining γ_0 as an algebraic unknown in whose terms $\bar{\sigma}$ and $\tilde{\sigma}$ may be stated (since given $p_i = 120$ MPa, $p_0 = 45$ MPa hence apply (2)). But we have developed design equations in the Notes, so may as well use them. Eg. (viii) in which three possibilities exist.

Since $p_0/p_i < 1$ then first possibility is no good

Since $\sigma^* \neq p_i$ then second possibility is out.

So try the first equation

$(\gamma_0 - 1)\sigma^* = 2\gamma_0(p_i - p_0)$ i.e. $(\gamma_0 - 1)160 = 2(120 - 45)\gamma_0$

Whence $\gamma_0 = 16$

– check that cited limits are valid

$0 \leq p_i/p_0 = 45/120 = 0.375 \leq (1 + 1/\gamma_0)/2 = \frac{17}{32} = 0.531$ – OK.

from (2) then $\bar{\sigma} = -40$ MPa $\tilde{\sigma} = 80$ MPa

Hence other stress/strain parameters as above.

Also $D_0/D_i = \sqrt{\gamma_0} = 4$ hence $D_i = D_0/4$.