Basic equations :-

$\bar{\sigma} = (p_i - \gamma_o p_o)/(\gamma_o - 1)$; $\tilde{\sigma} = (p_i - p_o)\gamma_o/(\gamma_o - 1)$

critical stress components at bore

$\sigma_t = \bar{\sigma} + \tilde{\sigma} = \dfrac{(\gamma_o+1)p_i - 2\gamma_o p_o}{(\gamma_o - 1)}$

$\sigma_r = \bar{\sigma} - \tilde{\sigma} = -p_i$ always

$\sigma_z = \bar{\sigma}$ (closed) $= (p_i - \gamma_o p_o)/(\gamma_o - 1)$

and for brittle the Modified Mohr Thy :

$1/n = \max(\hat{\sigma}/S_t, -\check{\sigma}/S_c, \hat{\sigma}/S_t - (\hat{\sigma}+\check{\sigma})/S_c)$

consider p_i fixed and examine the safety factor n as p_o increases from zero at first to infinity.

Initially $p_o = 0$

$\sigma_t = \dfrac{\gamma_o+1}{\gamma_o-1} \cdot p_i \qquad \sigma_r = -p_i \qquad \sigma_z = \dfrac{1}{\gamma_o - 1} \cdot p_i$

By inspection $\hat{\sigma} = \sigma_t$, $\check{\sigma} = \sigma_r$ and so

$1/n = \max\left[\dfrac{\gamma_o+1}{\gamma_o-1} \cdot \dfrac{p_i}{S_t} , \dfrac{p_i}{S_c} , \dfrac{\gamma_o+1}{\gamma_o-1} \cdot \dfrac{p_i}{S_t} - \dfrac{2}{\gamma_o-1} \cdot \dfrac{p_i}{S_c} \right]$

It may be shown that the first term is always larger than the other two, and so at $p_o = 0$

$S_t/n = \dfrac{\gamma_o+1}{\gamma_o-1} \cdot p_i$

As p_o starts to increase, the order of the stresses ($\hat{\sigma} = \sigma_t$, $\check{\sigma} = \sigma_r$) does not change

$\hat{\sigma} = \sigma_t = ((\gamma_o+1)p_i - 2\gamma_o p_o)/(\gamma_o - 1)$

$\check{\sigma} = \sigma_r = -p_i$

& $\hat{\sigma} + \check{\sigma} = 2(p_i - \gamma_o p_o)/(\gamma_o - 1)$

So $\dfrac{1}{n} = \max\left[\dfrac{(\gamma_o+1)p_i - 2\gamma_o p_o}{(\gamma_o-1)S_t} , p_i/S_c , \dfrac{(\gamma_o+1)p_i - 2\gamma_o p_o}{(\gamma_o-1)S_t} - \dfrac{2(p_i - \gamma_o p_o)}{(\gamma_o-1)S_c} \right]$

Arguing from initial conditions ($p_o = 0$), the first term remains the max. until it falls below the last term, i.e. when $p_i - \gamma_o p_o = 0$. So,

for $0 \le p_o/p_i \le 1/\gamma_o$

$(\gamma_o - 1)S_t/n = (\gamma_o+1)p_i - 2\gamma_o p_o$

As p_o further increases beyond p_i/γ_o , the third term becomes the maximum, i.e.

$\dfrac{1}{n} = \dfrac{(\gamma_o+1)p_i - 2\gamma_o p_o}{(\gamma_o-1)S_t} - \dfrac{2(p_i - \gamma_o p_o)}{(\gamma_o-1)S_c}$

or, setting $m \equiv S_c/S_t$

$(\gamma_o - 1)S_c/n = ((\gamma_o+1)m - 2)p_i - 2(m-1)\gamma_o p_o$

which is valid until exceeded by