

6 $r_0 = (240/80)^2 = 9$ - closed, so $\sigma_a = \bar{\sigma}$ ($\sigma_r = 0$),
 from (2) $\bar{\sigma} = (60 - 9 \times 20)/8 = -15$ MPa ; $\bar{\sigma} = (60 - 20)9/8 = 45$ MPa.
 from (3) $\sigma_t = -15 + 45/r$ $\sigma_a = -15$ $\sigma_r = -15 - 45/r$; $r = (D/80)^2$
 eg. $D = D_i = 80$ $r = 1$ $\sigma_t = 30$ $\sigma_a = -15$ $\sigma_r = -60$ ($= -r_i \checkmark$)
 $D = D_m = 160$ $r = 4$ -4 -15 -26
 $D = D_o = 240$ $r = 9$ -10 -15 -20 ($= -r_o \checkmark$)

The shape of the stress plot is identical to that shown in Notes - a factor of exactly 2 applies.

σ^* at bore = $\hat{\sigma} - \check{\sigma} = 30 - (-60) = 90$ MPa.

7 Basic equations:

$\hat{\sigma} = (p_i - r_0 r_o) / (r_o - 1)$; $\check{\sigma} = \frac{(r_i - r_o) r_o}{r_o - 1}$

at critical bore:
 $\sigma_t = \hat{\sigma} + \check{\sigma} = \frac{(r_o + 1)p_i - 2r_0 r_o}{r_o - 1}$

$\sigma_r = \hat{\sigma} - \check{\sigma} = -p_i$ always

$\sigma_a = 0$ since open

& $\sigma_e = \hat{\sigma} - \check{\sigma}$ - max. shear stress thry.
 Consider p_i fixed and examine what happens as r_0 increases from an initial value of zero.

Initially $r_0 = 0$ $\hat{\sigma} = \sigma_t = \frac{r_o + 1}{r_o - 1} p_i$; $\check{\sigma} = \sigma_r = -p_i$
 $\therefore \sigma_e = \hat{\sigma} - \check{\sigma} = \frac{2r_o}{r_o - 1} \cdot p_i$

As r_0 increases from 0, the tensile σ_t tends to become compressive. However σ_t remains $\hat{\sigma}$ provided $(r_o + 1)p_i \geq 2r_0 r_o$

So in this range $0 \leq r_0/p_i \leq (r_o + 1)/2r_o$
 $\sigma_e = \hat{\sigma} - \check{\sigma} = \sigma_t - \sigma_r = 2r_0 (p_i - r_0) / (r_o - 1)$

When r_0 increases beyond $\frac{r_o + 1}{2r_o} \cdot p_i$, σ_t does become compressive, i.e. $< \sigma_a$.

So $\hat{\sigma} = \sigma_a = 0$ and
 $\check{\sigma} = \sigma_r = -p_i$ still, provided $\sigma_t > \sigma_r$
 i.e. provided $r_0 \leq r_i$

In the range $\frac{r_o + 1}{2r_o} \leq r_0/p_i \leq 1$ therefore,
 $\sigma_e = \hat{\sigma} - \check{\sigma} = \sigma_a - \sigma_r = p_i$

Finally when $r_0 \geq r_i$, σ_t replaces σ_r as the minimum and
 $\sigma_e = \hat{\sigma} - \check{\sigma} = \sigma_a - \sigma_t = \frac{2r_0 r_o - (r_o + 1)p_i}{r_o - 1}$

Q.E.D.