1. Let $\sigma_o = \Delta_p D_i/4t$ – then, from (i)

$\sigma_t = 2\sigma_o$; $\sigma_a = 0$ (open) or σ_o (closed) ; $\sigma_r = 0$

– hence the two Mohr's circles in Notes.

Max. shear stress $\hat{\sigma} = \sigma_t = 2\sigma_o$ $\check{\sigma} = 0$ (for both open & closed)

$\therefore \sigma_E = \hat{\sigma} - \check{\sigma} = 2\sigma_o = \Delta_p D_i/2t$ & $= S_y/n$

Distortion energy – for plane stress ($\sigma_r = 0$), $\sigma_E^2 = \hat{\sigma}^2 - \hat{\sigma}\check{\sigma} + \check{\sigma}^2$

open : $\hat{\sigma} = 2\sigma_o$, $\check{\sigma} = 0$ $\therefore \sigma_E = 2\sigma_o$ – as previous case

closed : $\hat{\sigma} = 2\sigma_o$, $\check{\sigma} = \sigma_o$ $\therefore \sigma_E = \sqrt{3}\sigma_o = \sqrt{3}\Delta_p D_i/4t$ QED

Brittle – modified Mohr – $\check{\sigma}$ and $\hat{\sigma}$ as max. shear stress.

Using failure algorithm, $n = S_{ut}/\hat{\sigma}$.

$\therefore n\Delta_p D_i/t = 2 S_{ut}$. QED.

2. The design stress is $S = S_y/n = 110$ MPa.

The thin cylinder equations (ii) will be modified since $D_o = D_i + 2t$ is tabulated for the tubes; thus :-

$S = \Delta_p D_i/2t = \Delta_p(D_o - 2t)/2t = \Delta_p(D_o/2t - 1)$, or

$t = D_o/2(1 + S/\Delta_p)$ and, here, $t = D_o/2(1 + 110/20) = D_o/13$

So, determine minimum necessary thickness from this, then select next larger available thickness from table:-

D_o	60.3	76.1	88.9	101.6	114.3	139.7	165.1
t minimum $= D_o/13$	4.6	5.9	6.8	7.8	8.8	10.7	12.7
t available	4.9	5.9	9.5	9.5	9.5	NA	NA

The two largest sizes are not available in the necessary thickness. The 88.9 & 101.6 sizes are so thick that the material is used inefficiently, so from a stress point of view, use the 60.3 x 4.9 or 76.1 x 5.9 or 114.3 x 9.5

But of course there are other considerations such as pressure drop etc.

3 a) First find maximum equivalent stress accurately via thick cylinder equation (viii; max. shear stress; steel)

$\gamma_o = (D_o/D_i)^2 = [(D_i + 2t)/D_i]^2 = (1 + 2t/D_i)^2 = 1.08^2 = 1.166$

$\therefore \sigma^* = 2\Delta_p\gamma_o/(\gamma_o - 1)$ or $\sigma^*/\Delta_p = 2\gamma_o/(\gamma_o - 1) = 14.0$

Now find σ_E for given Δ_p from thin cylinder approx :-

diameter basis, D	D_i	D_m	D_o
D/t for $D_i/t = 25$	25	26	27
$\sigma_E/\Delta_p = D/2t$	12.5	13	13.5
error $= 1 - \sigma_E/\sigma^*$ %	10.8	7.3	3.7

Thus these other formula certainly reduce the error.

b) If the thin cylinder equation used is $\sigma_E = \Delta_p D_m/2t$

in which $D_m = \frac{1}{2}(D_i + D_o) = D_i + t$

and $\Delta_p = p_i$ since internally pressurised

then $\sigma_E = p_i(D_i + t)/2t$

Or, solving for t, the usual required unknown, $t = \dfrac{D_i}{2\sigma_E/p_i - 1}$