Inserting boundary conditions:
$v = 0$ @ $\rho = 0$ & $v' = 0$ @ $\rho = \frac{1}{2}$
leads to:

$$v = \frac{QL}{P}\left\{ \frac{\sin \pi\sqrt{\psi}\cdot\rho}{\pi\sqrt{\psi}\cdot\cos\frac{\pi}{2}\sqrt{\psi}} - \rho \right\} \quad \text{— QED.}$$

so $\hat{v}$ @ $\rho = \frac{1}{2}$: $\hat{v} = \frac{QL}{2P}\left[\frac{\tan\frac{\pi}{2}\sqrt{\psi}}{\frac{\pi}{2}\sqrt{\psi}} - 1 \right]$

$\therefore$ $\hat{M}$ @ $\rho = \frac{1}{2}$: $\hat{M} = Q.\frac{1}{2}L + P\hat{v}$

$$= \frac{QL}{2}\cdot\frac{\tan\frac{\pi}{2}\sqrt{\psi}}{\frac{\pi}{2}\sqrt{\psi}}$$

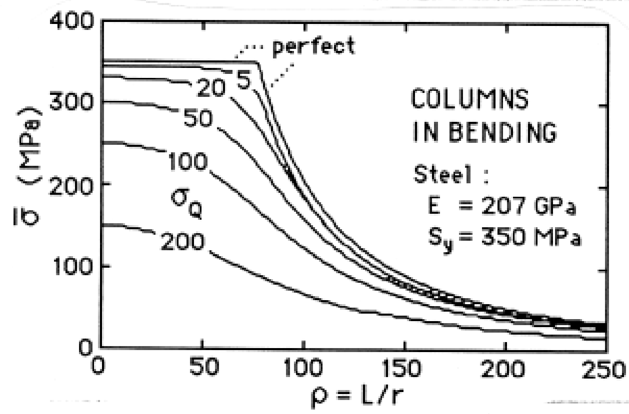
and so max. stress

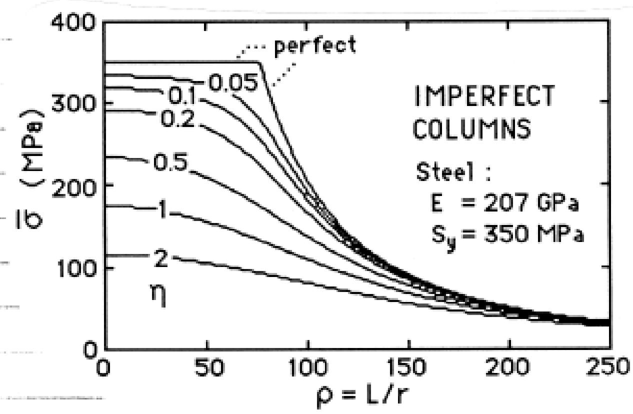
$$\hat{\sigma} = \frac{\hat{M}y}{I} + \frac{P}{A} = \frac{QLy}{2I}\cdot\frac{\tan\frac{\pi}{2}\sqrt{\psi}}{\frac{\pi}{2}\sqrt{\psi}} + \frac{P}{A}$$

$\frac{QLy}{2I}$ is just the max. bending stress due to Q alone so applying this to the limiting case when P is the critical load required to induce $\hat{\sigma} \rightarrow S$,

$$S = \sigma_Q\cdot\frac{\tan\frac{\pi}{2}\sqrt{\psi}}{\frac{\pi}{2}\sqrt{\psi}} + \bar{\sigma} \qquad \text{Q.E.D.}$$

To plot $\bar{\sigma}$ vs ρ, we must solve this eq. iteratively, since $\psi = \bar{\sigma}/\sigma_c$. The resulting graphs are as shown :-





Transverse loading of columns can evidently be likened to equivalent imperfections.