7(concl'd) Repeating this with σ_η given, rather than η, equation (3) is applicable.

$$\bar{\sigma}^2 - \bar{\sigma}[S + \sigma_c + \sigma_\eta] + S\sigma_c = 0$$

which, with $S = 250$, $\sigma_c = 491$, $\sigma_\eta = 60$ MPa, gives $\bar{\sigma} = 206$

$\therefore \quad F = A\bar{\sigma}/n = 1510 \times 206/2 = 156$ kN.

Note how much simpler this approach is in that $\hat{y}$ e.g. does not need to be found.

8.(a) Equation (2) derives from equation (g) which refers to the maximum compressive stress on the concave side at the column (pin-ended) mid-length. Although the maximum tensile stress on the convex side (if tension exists) must be less than the compressive maximum, for a brittle material the tensile strength S_t is also less than the compressive strength S_c. So the safety factor based on tension must be examined.

Reformulating (g), the maximum tensile stress is:

$$\hat{\sigma} = \hat{\sigma}_{bending} - \sigma_{direct}$$

i.e. $1/\theta = \eta(\hat{\omega}/\hat{e} + 1) - 1$, and from (f)(iii)

$= \eta/(1 - \phi) - 1$

Substitute $\theta = \bar{\sigma}/\hat{\sigma} \Rightarrow \bar{\sigma}/S_t$ and $\phi = \bar{\sigma}/\sigma_c$ gives

$$(\sigma_c - \bar{\sigma})(S_t + \bar{\sigma}) = \eta\sigma_c\bar{\sigma} \quad \text{or}$$

(A) $\quad \bar{\sigma}^2 + \bar{\sigma}[S_t - (1-\eta)\sigma_c] - S_t\sigma_c = 0$

Tensile considerations predominate if $\bar{\sigma}$, as given by (A), is less than $\bar{\sigma}$ as given by (2). So (A) applies if:

$$-[S_t - (1-\eta)\sigma_c] + \sqrt{\{[S_t - (1-\eta)\sigma_c]^2 + 4S_t\sigma_c\}} <$$
$$[S_c + (1+\eta)\sigma_c] - \sqrt{\{[S_c + (1+\eta)\sigma_c]^2 - 4S_c\sigma_c\}}$$

which, after some manipulation, simplifies to:

(B) $\quad 1/(S_c - S_t) - \eta/(S_c + S_t) < 1/2\sigma_c$

So, if (B) is satisfied then (A) applies, otherwise (2) [with $S \equiv S_c$] applies.

(b) $k_L = 0.7$ (end-fixed) so $L = 0.7 \times 400 = 280$ mm.

$A = 20^2 = 400$ mm^2 $\quad I = \frac{1}{12} \times 20^4 \quad \therefore r = 10/\sqrt{3}$ mm.

From text tables for a 765 MPa compressive strength cast iron, $S_t = 220$ MPa and $E = 100$ GPa.

$\therefore \sigma_c = E(\pi/\rho)^2 = E(\pi/L)^2 r^2 = (100 \times 10^3)(\pi/280)^2 \times \frac{100}{3} = 420$ MPa.

From (B) above, with $\eta = 1$

$\therefore 1/(S_c - S_t) - \eta/(S_c + S_t) = 1/545 - 1/985 < 1/840 = 1/2\sigma_c$

$\therefore$ Tensile considerations predominate and (A) is applicable. From (A), $\bar{\sigma} = 213$ MPa

$\therefore \bar{F}\,[n=1] = A\bar{\sigma} = 400 \times 213 = \underline{85}$ kN