First derive expression for sensitivity of pivoted shoe given μ and the geometry – r a b_x b_y and the vectors of lining integrals J_s and J_c.

For simplicity, define $\zeta = \Delta\delta\mu$ & let $' \equiv d/d\zeta$

(a) $\begin{cases} \alpha = [\,\zeta a \quad a \quad -\zeta r\,] & \alpha' = [\,a \quad 0 \quad -r\,] \\ \beta = [\,(\zeta b_x - b_y) \quad (\zeta b_y + b_x) \quad -\zeta r\,] & \beta' = [\,b_x \quad b_y \quad -r\,] \end{cases}$

Define also the ratio ν between lining press. components

(b) $\nu \equiv N_c/N_s = p_c/p_s \quad \Delta = -\beta J_s/\beta J_c$ from (11)

and form the vector J_o via:

(c) $J_o = J_s + \nu J_c$

– noting that $\alpha, \alpha', \beta, \beta', \nu, J_o$ are all computable given the known geometry, as is:

$\nu' = -\frac{d}{d\zeta}(\beta J_s/\beta J_c) = -(\beta' J_s/\beta J_c - \beta J_s.\beta' J_c/(\beta J_c)^2)$

(d) $= -(\beta' J_s + \nu\beta' J_c)/\beta J_c = -\beta' J_o/\beta J_c$

In this J_o notation, equations (5c) & (7c) take the form:–

(e) $\begin{bmatrix} F_x \\ F_y \end{bmatrix} = N_s \begin{bmatrix} \Delta & -\delta\mu \\ \delta\mu & \Delta \end{bmatrix} \begin{bmatrix} I_{sc} + \nu I_{cc} \\ I_{ss} + \nu I_{sc} \end{bmatrix} = N_s \begin{bmatrix} \Delta & -\delta\mu \\ \delta\mu & \Delta \end{bmatrix} \begin{bmatrix} J_o(1) \\ J_o(2) \end{bmatrix}$

(f) $|T| = \mu r N_s (I_s + \nu I_c) = N_s.\mu r J_o(3)$

(g) $M = N_s \alpha (J_s + \nu J_c) = N_s.\alpha J_o$

The mechanical advantage, η is

$\eta = |T|/M = \mu r J_o(3)/\alpha J_o$ and so

$d\eta/d\mu = r\{J_o(3)/\alpha J_o + \zeta(\,(J_o(3))'/\alpha J_o - J_o(3)(\alpha J_o)'/(\alpha J_o)^2\,)\}$

$= \frac{\eta}{\mu}\left(1 + \zeta\left(\frac{(J_o(3))'}{J_o(3)} - \frac{(\alpha J_o)'}{\alpha J_o}\right)\right)$

– and as the sensitivity, from (vii) is:

(h) $S = 1 + \zeta(\,(J_o(3))'/J_o(3) - (\alpha J_o)'/\alpha J_o\,)$

in which

(m) $(J_o(3))' = \frac{d}{d\zeta}(I_s + \nu I_c) = \nu' I_c = \nu' J_c(3)$

(n) $(\alpha J_o)' = \frac{d}{d\zeta}(\alpha J_o) = \alpha' J_o + \alpha J_o'$

$= \alpha' J_o + \alpha(J_s + \nu J_c) = \alpha' J_o + \nu' \alpha J_c$

All terms herein have been defined, hence S.

Calculating parameters for the LH shoe, $\Delta\delta = 1$, $\zeta = 0.3$

given geometry:

$r = 10''$ $a = 12.37''$ $b_x = 11\cos 80° = 1.91''$ $b_y = 11 \sin 80° = 10.83''$

$\therefore$ from (a) $\alpha = [\,3.71 \quad 12.37 \quad -3\,]$ in, $\alpha' = [\,12.37 \quad 0 \quad -10\,]$ in

$\beta = [\,-10.26 \quad 5.16 \quad -3\,]$ in, $\beta' = [\,1.91 \quad 10.83 \quad -10\,]$ in

and the integral vectors:–

$J_s = [\,0.236 \quad 1.436 \quad 1.714\,]$ $\quad J_c = [\,0.833 \quad 0.236 \quad 0.590\,]$

Applying (b):

$\left.\begin{aligned} \beta J_c &= [-10.26 \quad 5.16 \quad -3][0.833 \quad 0.236 \quad 0.590] = -9.096 \\ \beta J_s &= [-10.26 \quad 5.16 \quad -3][0.236 \quad 1.436 \quad 1.714] = -0.150 \end{aligned}\right\} \nu = \underline{-0.0165}$

So, from (c) $J_o = \begin{bmatrix} 0.236 - 0.0165 \times 0.833 \\ 1.436 - 0.0165 \times 0.236 \\ 1.714 - 0.0165 \times 0.590 \end{bmatrix} = \begin{bmatrix} 0.222 \\ 1.432 \\ 1.704 \end{bmatrix}$