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Assume $y_1 = a + bx$, $y_2 = \alpha + \beta x$ where a, b, α, β are const.

Then $z^2 = (a + bx)^2 + \lambda^2 (\alpha + \beta x)^2$

$$\frac{1}{2} \frac{d(z^2)}{dx} = (2b + \lambda^2 \alpha / \beta) + (b^2 + \lambda^2 \beta^2) x \quad (1)$$

$$\frac{1}{2} \frac{d^2(z^2)}{dx^2} = b^2 + \lambda^2 \beta^2 > 0 \quad (2)$$

So z^2 may have a turning value if (1) is zero.

But the turning value must be a minimum via (2).

Therefore any maximum value of z^2 , and hence of z , must occur at the ends of the interval.

This useful result is applicable to a beam e.g. when moment diagrams in the vertical & horizontal planes might be so sketched. Then maximum net



bending moment, $M = \sqrt{(M_v^2 + M_h^2)}$, must occur at loads. The same reasoning is applicable to $M_E = \sqrt{(M^2 + T^2)}$ or $= \sqrt{(M^2 + \frac{3}{4} T^2)}$ in eq (11). The search is greatly reduced.