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consider element on surface where σ_b, τ_t are known to be maxima.

The loading is as shown (Note how this differs from thin cylinder + M + T).

Clearly the z-direction is principal, however

we'll have to resolve in x-y plane in order to find all three principals, prior to implementing the failure criterion.

The stress state is thus:

$$\sigma_x = M/z \quad \sigma_y = -p \quad \tau_{xy} = T/2z \quad \sigma_z = -p \text{ (prin.)}$$

In the x-y plane:

$$\bar{\sigma} = \frac{1}{2}(M/z - p)$$

& base length of Δ under O-x:

$$= M/z - \bar{\sigma} = \frac{1}{2}(M/z + p)$$

$$\therefore \sigma_2 = \sqrt{\left[\frac{1}{2}(M/z + p)\right]^2 + (T/2z)^2}$$

$$= \frac{1}{2}\sqrt{(M/z + p)^2 + (T/z)^2}$$

In the x-y plane still,

$$\sigma_1 = \bar{\sigma} + \sigma_2 = \frac{1}{2}\left[\frac{M}{z} - p + \sqrt{(M/z + p)^2 + (T/z)^2}\right]$$

$$\sigma_3 = \bar{\sigma} - \sigma_2 = \frac{1}{2}\left[\frac{M}{z} - p - \sqrt{(M/z + p)^2 + (T/z)^2}\right]$$

(A)

From the 3-Mohr's circles, clearly $\sigma_2 \neq \bar{\sigma}$ since

$$\sigma_2 = \sigma_y = -p.$$

That is $\bar{\sigma}$ is minimum of 3 principals.

Applying max. shear stress theory:

$$\sigma_e = \hat{\sigma} - \hat{\sigma} \quad (\hat{\sigma}, \hat{\sigma} \text{ max, min 3-D stresses})$$

$$= \sqrt{(M/z + p)^2 + (T/z)^2} \quad \text{from (A) above}$$

So the shaft design equation is

$$S/n = \sigma_e, \text{ or}$$

$$n \sqrt{(M/z + p)^2 + (T/z)^2} = S$$

- which degenerates to eq. (11b) of Note, when $p \rightarrow 0$

The bottom of the shaft should also be checked

