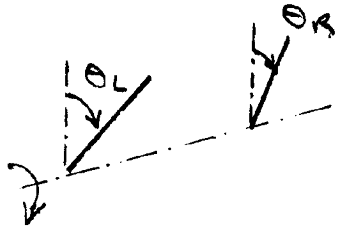
8 cont'd TORSION

Compatibility

If θ_L and θ_R are the angular deflexions of the tube in way of L and R discs, the rotation of the tube (relative) between the discs, θ_t, is

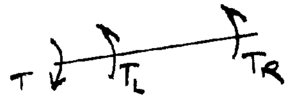
$$\theta_t = \theta_L - \theta_R .$$



Equilibrium

The stress resultants in the bonds are T_L & T_R, where

$$T_L + T_R = T = 10 \text{ Nm}$$



Note that the relative rotation of the tube between the discs is due to the torque transmitted along the tube between them, that is T_R as may be seen by a RH free body.

Constitutive Laws.

Discs : $T_L = k_d \theta_L$; $T_R = k_d \theta_R$

where k_d is the disc stiffness

$$k_d = \frac{4\pi b G}{1/r_i^2 - 1/r_o^2} = \frac{4\pi \times 20 \times 2}{2(1+0.5)\{1/60^2 - 1/100^2\}} 10^{-3} = 942 \text{ Nm/rad.}$$

$$\text{mm} \quad \frac{\text{N}}{\text{mm}^2} \quad \text{mm}^2 \quad \frac{\text{m}}{\text{mm}}$$

Tube : from torsion eqn.

$$k_t = \frac{T}{\theta} = \frac{GJ}{L} = \frac{2E3}{2(1+0.35)} \frac{\frac{\pi}{32}(60^4 - 54^4)}{600 \times 10^3} = 540 \text{ Nm/rad.}$$

$$T_R = k_t \theta_t \qquad \frac{\text{N}}{\text{mm}^2} \quad \text{mm}^4 \frac{1}{\text{mm}} \frac{\text{m}}{\text{mm}}$$

Solution $T_L = 7.3$ Nm ; $T_R = 2.7$ Nm

In terms of the shear stress in the bond, the torsional stress resultant is

$$T\,(=T_L \text{ or } T_R) = \tau A r = \tau \times 2\pi r^2 b \quad ; r = 30 \text{ mm.}$$

So the tangential shear components are

$$\tau_L = 65 \text{ kPa} \qquad \tau_R = 24 \text{ kPa.}$$

Evidently the left bond is much the more heavily loaded. For this, from above

$\tau_A = 60$ kPa $\quad \tau_C = 65$ kPa

$\therefore \tau = \sqrt{(60^2 + 65^2)} = 95$ kPa

and the safety factor for the bonds is

$n = \hat{\tau}/\tau = 100/95 = \underline{1.05}$.