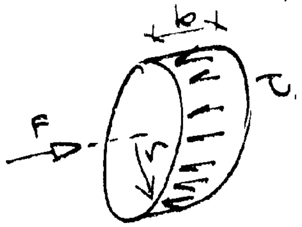
7 Approach – assume shear stress τ is constant at general radius r ($r_i \le r \le r_o$), hence evaluate it by equilibrium with external load. Knowing τ, we may find shear strain, γ, assuming elastic behaviour. This may be integrated over r to find the deflexion under the load. Hence stiffness.

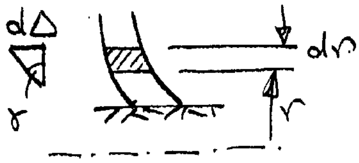


AXIAL FORCE – F.

For equilibrium $F = 2\pi r b \tau$

$\therefore \gamma = \tau / G = (F/2\pi b G).\frac{1}{r}$.

Shear strain is thus inversely proportional to radius, and the deformed shape will be:



If Δ is the axial deformation

$\gamma = d\Delta / dr$, or

$d\Delta = \gamma . dr = (F/2\pi b G)\frac{1}{r}\,dr$

Integrating:

$\Delta = (F/2\pi b G)\ln r + \text{constant}$.

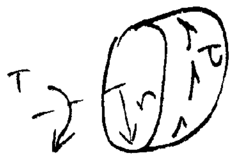
Inserting limits $\Delta = 0$ at $r = r_i$, $\Delta = \Delta_{total}$ at $r = r_o$

$\Delta_{total} = (F/2\pi b G)\ln r_o/r_i$.

and so stiffness is

$k = F/\Delta_{total} = 2\pi b G / \ln\frac{r_o}{r_i}$

TORQUE – T



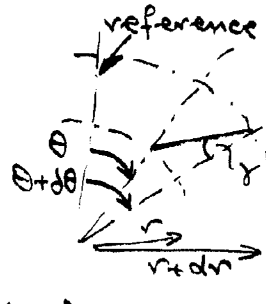
For equilibrium $T = 2\pi r^2 b \tau$

$\therefore \gamma = \tau / G = (T/2\pi b G).1/r^2$

The increase in torsional deformation, θ, is:–

$r\,d\theta = \gamma\,dr = \frac{T}{2\pi b G}\frac{1}{r^2}dr$

i.e. $d\theta = (T/2\pi b G)\,r^{-3}\,dr$



Integrating between the limits

$\theta = 0$ at $r = r_i$ & $\theta = \theta_{total}$ at r_o

$\theta_{total} = (T/4\pi b G)(1/r_i^2 - 1/r_o^2)$

Hence the torsional stiffness

$k = \frac{T}{\theta_{total}} = 4\pi b G / (1/r_i^2 - 1/r_o^2)$

It should be realised that the foregoing treatment is a gross simplification of an extremely complex problem. However it enables a first order, "ball park" estimate to be made.