

3 (cont'd)

Compatibility Laws. Effective length of 1 & 2 is 250 mm

Beet. tensile force F_1 & extension δ_1
 $F_1 = k_1 \delta_1$; $k_1 = \frac{\pi}{4} \times 18^2 \times 207 / 250 = 211 \text{ kN/mm}$

Sleeve compressive F_2 & contraction δ_2
 $F_2 = k_2 \delta_2$; $k_2 = \frac{\pi}{4} (35^2 - 20^2) \times 50 / 250 = 130 \text{ kN/mm}$

Solution - 4 equations, 4 unknowns.

E - eliminate deflections to get

$$\left. \begin{aligned} F_1 &= k_e (\Delta + P/k_2) = F_i + P k_e / k_2 \\ F_2 &= k_e (\Delta - P/k_1) = F_i - P k_e / k_1 \end{aligned} \right\} \frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

where F_i is the initial load (i.e. $P=0$) due to tightening by Δ .

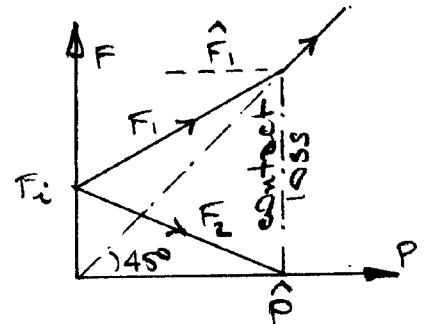
$$k_e = 1 / \left(\frac{1}{211} + \frac{1}{130} \right) = 80 \text{ kN/mm}$$

So the force relations are:

$$F_1 = F_i + 0.619 P$$

$$F_2 = F_i - 0.381 P$$

These plots as shown. As P increases, the load in the beet increases also, but that in the sleeve drops from initial F_i until eventually contact is lost and F_1 takes all P .



To increase the load \hat{P} , when contact is lost, we therefore have to increase F_i . But this tends to increase both F_1 and F_2 . The maximum forces sustainable are

$$\hat{F}_1 = \frac{\pi}{4} \times 18^2 \times 550 = 140 \text{ kN}$$

$$\hat{F}_2 = \frac{\pi}{4} (35^2 - 20^2) \times 80 = 52 \text{ kN.}$$

From above therefore

if F_1 is limiting, $\max \hat{P} = 140 \text{ kN.}$

if F_2 is limiting, $\max \hat{P} = 52 \text{ kN}$

which latter corresponds to $\hat{P} = 52 / 0.381 = 136 \text{ kN}$

So the sleeve limits the load to 136 kN, which necessitates an initial force of 52 kN.

4. If the bars share the load, their required cross-sectional areas are each

$$A = 5E3(N) \times 2.5 / 250(N/mm^2) = 50 \text{ mm}^2$$

and so stiffness is $k = 50 \times 207 / 500 = 20.7 \text{ kN/mm}$

Indeterminate, let "c" subscript refer to centre bar, "o" subscript refer to each of outer bars.

Compatibility

Assume centre bar is long by amount Δ .