12 $d^* = (1/d_1 + 1/d_2)^{-1} = (1/10 - 1/10.1)^{-1} = 1010$ mm ← since internal.

$E^* = \left(\frac{1-.29^2}{207} + \frac{1-.3^2}{110}\right)^{-1} = 78.8$ GPa.

For spheres: $a^3 = \frac{3Fd^*}{8E^*}$ $\quad \hat{p} = \frac{3F}{2\pi a^2}$

i.e. the interdependence of $\hat{p}$ & F is non-linear so eliminate 'a' to obtain direct relation

$a^6 = 9F^2 d^{*2}/64E^{*2} = 27F^3/8\pi^3\hat{p}^3$

whence: $F = \frac{1}{24}(\pi\hat{p})^3(d^*/E^*)^2$ (i)

(a) Using equation (i) above, with $\hat{p} = 200$ MPa

$\hat{F} = \frac{1}{24}(\pi \times 200)^3 (1010/78.8\times10^3)^2 = 1.70$ kN

$\quad N^3/mm^6 \quad mm^2 \quad mm^4/N^2$

(b) Both materials are ductile, so max. shear criterion is valid, i.e. $\hat{\sigma}_e = 0.6\hat{p}$

bronze $\hat{\sigma}_E = 0.6\hat{p} \le 240 \quad \therefore \hat{p} \le 400$ MPa.

steel $\hat{\sigma}_E = 0.6\hat{p} \le 400 \quad \therefore \hat{p} \le 667$ MPa

Obviously bronze is weaker & $\hat{p} = 400$ MPa

Since $F \propto \hat{p}^3$, and $\hat{p}$ is double that of (a), then

$F = 2^3 \times 1.70 = 13.6$ kN

(c) $E^* = [1-.29^2/207 + 1-.21^2/100]^{-1} = 71.5$ GPa.

From above, if steel is weaker material at the contact, $\hat{p} = 667$ MPa.

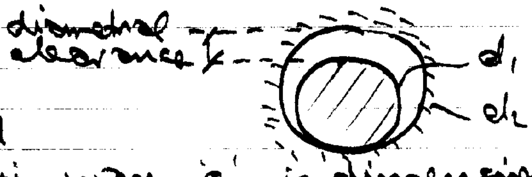
Considering the cast iron, if it fails, it will do so by direct compressive stress on the surface exceeding the compressive design stress. I.e. if C.I. is weaker then $\hat{p} \le 350$ MPa.

So C.I. is in fact the weaker, so $\hat{p} = 350$ MPa

& $F = \frac{1}{24}(\pi \times 350)^3 (1010/71.5\times10^3)^2 = 11.0$ kN

13 $d_1 \approx d = d(1 - \frac{1}{2}c)$

$d_2 \approx d = d(1 + \frac{1}{2}c)$



i.e. clearance $= d_2 - d_1 = cd$

noting that, expressed this way, 'c' is dimensionless

So $d^* = (1/d_1 + 1/d_2)^{-1}$, $d_2 < 0$ since internal

$= (1/d(1-\frac{1}{2}c) - 1/d(1+\frac{1}{2}c))^{-1} = d/c$

– Note the implication of improving the fit.

Eliminate unwanted 'a' from cylinder equations

$a^2 = 2Fd^*/\pi L E^*$ $\quad \hat{p} = 2F/\pi a L$

gives, $\hat{p} = \sqrt{\left[\frac{2}{\pi}\cdot\frac{F}{Ld^*}\cdot E^*\right]}$

Note the implication of this, $\hat{p}$ is approx. the geometric mean of E^* and average pressure over the contact F/Ld^*

Finally substituting for d^* ⇒ $\hat{p} = \sqrt{(2cFE^*/\pi d L)}$