8 . Assume dummy load of $P = 1$ N.

$\therefore$ from equil^{m} : $F = 1$ N

& $M = -1 \times (5.2 - 2.1) = -3.1$ N mm

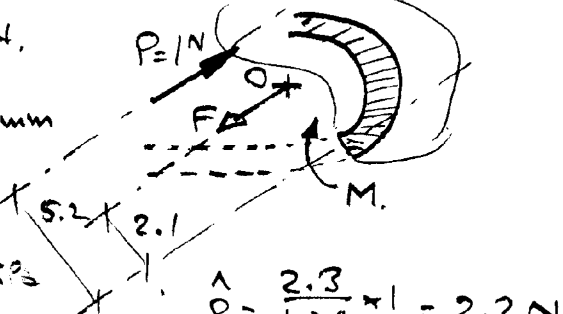
Section properties :-

$r_c = 2.1 \quad r_i = 1.91 \quad r_o = 2.29$

$\therefore r_d = \frac{1}{4}(\sqrt{1.91} + \sqrt{2.29})^2 = 2.0957$ mm

from (3): $r_n = 2.0974 \quad \bar{\sigma} = -10.6$ GPa

$\therefore$ at r_i, $\sigma = 1.04$ GPa



$\therefore \hat{P} = \frac{2.3}{1.04} \times 1 = \underline{2.2\text{ N}}$

9 The vectors are defined as :

$r \equiv [x \ y \ 0]'$ — $z = 0$ since δA in x-y plane.

$b \equiv [b_x \ b_y \ b_z]'$ unit stress MPa/mm.

We start with the given equation

$$M \equiv [M_x \ M_y \ M_z]' = \int_A r \times (b \times r)\, dA.$$

Forming the triple product prior to integration:

$$b \times r = \begin{vmatrix} i & j & k \\ b_x & b_y & b_z \\ x & y & 0 \end{vmatrix} = i(-y b_z) + j(x b_z) + k(y b_x - x b_y)$$

$$\text{i.e.} \equiv [-y b_z \quad x b_z \quad (y b_x - x b_y)]'$$

$$r \times (b \times r) = \begin{vmatrix} i & j & k \\ x & y & 0 \\ -y b_z & x b_z & (y b_x - x b_y) \end{vmatrix} = \begin{bmatrix} b_x y^2 - b_y x y \\ b_y x^2 - b_x x y \\ b_z (x^2 + y^2) \end{bmatrix}$$

The integral thus takes the form :

$$M = \int_A \begin{bmatrix} b_x y^2 - b_y x y \\ b_y x^2 - b_x x y \\ b_z (x^2 + y^2) \end{bmatrix} dA = \begin{bmatrix} b_x \int y^2 dA - b_y \int x y\, dA \\ b_y \int x^2 dA - b_x \int x y\, dA \\ b_z \int (x^2 + y^2)\, dA \end{bmatrix}$$

or, making the normal substitutions :

$I_{xx} = \int y^2 dA \quad I_{yy} = \int x^2 dA \quad I_{xy} = \int x y\, dA \quad J_{zz} = \int (x^2 + y^2) dA$

$$M = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} b_x I_{xx} - b_y I_{xy} \\ b_y I_{yy} - b_x I_{xy} \\ b_z J_{zz} \end{bmatrix} = \begin{bmatrix} I_{xx} & -I_{xy} & 0 \\ -I_{xy} & I_{yy} & 0 \\ 0 & 0 & J_{zz} \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$$

$= I b$ as given in (vi)

The last of these three equations is irrelevant – it deals with torsion (i.e. $M_z \equiv T_z \rightarrow \tau_{zx}$ & τ_{zy}). So, concentrating on bending ($b_z = 0$)

$$\begin{bmatrix} M_x \\ M_y \end{bmatrix} = \begin{bmatrix} I_{xx} & -I_{xy} \\ -I_{xy} & I_{yy} \end{bmatrix} \begin{bmatrix} b_x \\ b_y \end{bmatrix} \quad \text{from above.}$$

Solving these two simultaneous equations for b:-

$$\left.\begin{aligned} b_x &= (M_x I_{yy} + M_y I_{xy}) / I_0^2 \\ b_y &= (M_x I_{xy} + M_y I_{xx}) / I_0^2 \end{aligned}\right\} I_0^2 = I_{xx} I_{yy} - I_{xy}^2$$

The stresses, from (V) with $c = 0$, are thus :

$$\sigma = b \times r = \begin{vmatrix} i & j & k \\ b_x & b_y & 0 \\ x & y & 0 \end{vmatrix} = \begin{bmatrix} 0 \\ 0 \\ b_x y - b_y x \end{bmatrix} \text{ i.e.} \equiv \begin{bmatrix} \tau_{zx} \\ \tau_{zy} \\ \sigma_z \end{bmatrix}$$

So the normal stress on δA at r is σ_z, or briefly $\sigma = [(M_x I_{yy} + M_y I_{xy}) y - (M_x I_{xy} + M_y I_{xx}) x] / I_0^2$