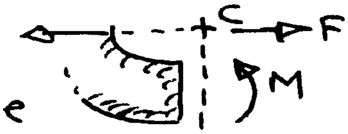
To the left of i-i curved beam theory applies.
section props: $r_i = R - r$ $\quad r_o = R + r$ $\quad r_c = R$

$2\sqrt{r_d} = \sqrt{r_o} + \sqrt{r_i}$

$\therefore\ 4 r_d = r_o + r_i + 2\sqrt{r_o r_i} = 2R + 2\sqrt{R^2 - r^2}$

$r_d = \frac{1}{2}R\left[1 + \sqrt{1-\lambda^2}\right] < R$ where $\lambda = r/R$

equil^m of end, stress resultant



$F = P \quad ; \quad M = 0$

so from (3) $\quad r_n = r_c = R$ from above

$\sigma_m = -F r_d / A(r_c - r_d)$

$= -P\frac{1}{2}R\left[1+\sqrt{1-\lambda^2}\right] / A\,\frac{1}{2}R\left[2 - 1 - \sqrt{1-\lambda^2}\right]$

$= -\sigma_d\left(1+\sqrt{1-\lambda^2}\right)/\left(1-\sqrt{1-\lambda^2}\right)$ where $\sigma_d = P/A$

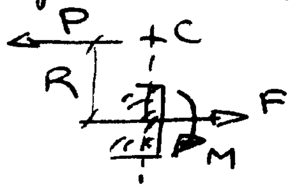
max. stress at inside, from (2):

$\hat{\sigma} = \sigma_m\left(1 - r_n/r_i\right) = -\sigma_d \frac{1+\sqrt{\ }}{1-\sqrt{\ }}\left(1 - \frac{R}{R-r}\right)$

(a) $\quad = \sigma_d \frac{\lambda}{1-\lambda}\cdot\frac{1+\sqrt{1-\lambda^2}}{1-\sqrt{1-\lambda^2}}$ the required result.

Note, as $\lambda \to 0$ (i.e. $\to$ straight) then
$\hat{\sigma}/\sigma_d \to 4/\lambda$ i.e. $\to \infty$.

To the right of the interface, for straight beam:
For equil^m M is clockwise, so
tensile bending stress at top is
additive to direct stress



$\hat{\sigma} = \sigma_d + \hat{\sigma}_b$ at top

$= F/A + M\hat{y}/I$

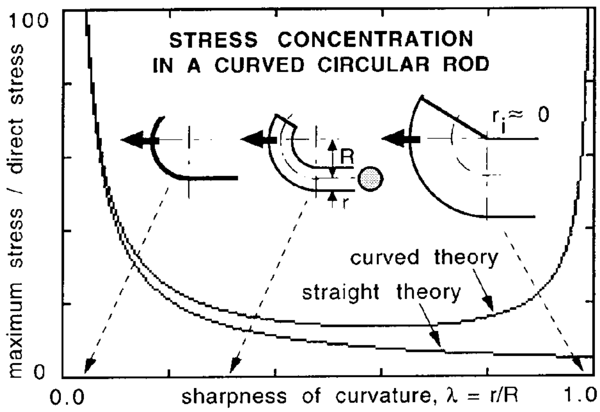
$= P/A + PR\,r / \frac{\pi}{4}r^4$

i.e. expressing as multiple of direct σ_d:

(b) $\quad \hat{\sigma} = \sigma_d(1 + 4/\lambda)$ where $\lambda = r/R$.

Note same asymptote as (a) when $\lambda \to 0$

(a) & (b) are plotted:- At low curvature ($R \gg r$, $\lambda \to 0$) both theories tend to same asymptote $\hat{\sigma}/\sigma_d = 4/\lambda$, i.e. curvature induced s.c. is negligible though $\sigma_b = \frac{4}{\lambda}\sigma_d \gg \sigma_d$.



$\hat{\sigma}_a$ is always $> \hat{\sigma}_b$, and s.c. due to curvature
is $\hat{\sigma}_a/\hat{\sigma}_b$ increases monotonically with λ.

When curvature is severe ($r_i \to 0$, $\lambda \to 1$) then $\hat{\sigma}_a \to \infty$, i.e. s.c. also $\to \infty$, though in practice yielding redistributes stress.