5 Section properties $r_i = 25$ $r_o = 63$ $b_i = 20$ $b_o = 6$ mm

$A = \frac{1}{2}(63-25)(20+6) = 494 \ \text{mm}^2$

$A r_c = \frac{1}{6}(63-25)\left[6(126+25) + 20(50+63)\right] = 20\,051 \ \text{mm}^3$ $\quad r_c = 40.59$ mm

$A/r_d = 6 - 20 + (20\times63 - 6\times25)\ln\frac{63}{25} / (63-25)$ $\therefore$ $r_d = 38.01$ mm

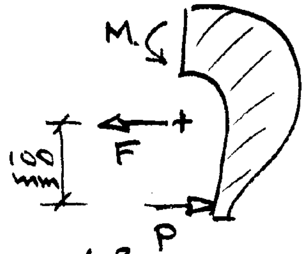
From equilibrium of arm:

$F = P = 2$ kN

$M = -P\times100 = -200$ Nm

From (3) $\quad r_n = 38.72$ mm, $\bar{\sigma} = -216$ MPa.

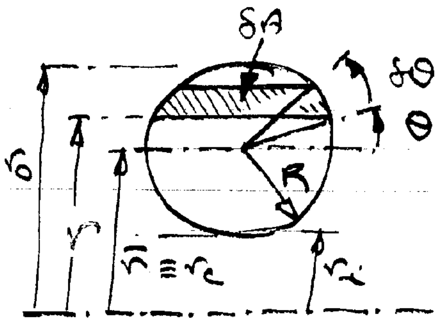
From (2) $\quad \sigma = -216\,(1 - 38.72/r)$ with r in mm



Hence:

r (mm)	25	30	40	50	60	63
σ (MPa)	118.6	62.8	−6.9	−48.8	−76.7	−83.3

6 Using (1) $\quad A/r_d = \int_{r_i}^{r_o} dA/r$



Assume element defined by θ, $\delta\theta$ as shown, so that δA and r under the integral are both functions of the same variable:–

$r = \bar{r} + R\sin\theta$

$\delta A = 2R\cos\theta * R\,\delta\theta\cos\theta = 2R^2\cos^2\theta\, d\theta.$

So the integral becomes:

$$\frac{\pi R^2}{r_d} = \int_{-\pi/2}^{\pi/2} \frac{2R^2\cos^2\theta\, d\theta}{\bar{r} + R\sin\theta}$$

Set $\lambda \equiv \bar{r}/R \geq 1$ temporarily for convenience; then

$$\frac{\pi R}{2 r_d} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2\theta\, d\theta}{\sin\theta + \lambda} = \int \frac{(1-\sin^2\theta)\, d\theta}{\sin\theta + \lambda}$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left\{ \lambda - \sin\theta - \frac{\lambda^2 - 1}{\sin\theta + \lambda} \right\} d\theta$$ see tables for integral

$$= \left[\lambda\theta + \cos\theta - \sqrt{\lambda^2-1}\ \arcsin\frac{1+\lambda\sin\theta}{\sin\theta+\lambda} \right]_{-\pi/2}^{\pi/2}$$

$$= \pi\left(\lambda - \sqrt{\lambda^2 - 1}\right)$$

But $\bar{r} = \frac{1}{2}(r_o + r_i)$; $R = \frac{1}{2}(r_o - r_i)$; $\lambda = \frac{r_o + r_i}{r_o - r_i}$ and

$$\frac{r_o - r_i}{4 r_d} = \frac{(r_o + r_i) - \sqrt{\left[(r_o+r_i)^2 - (r_o-r_i)^2\right]}}{r_o - r_i} = \frac{r_o + r_i - 2\sqrt{r_o r_i}}{r_o - r_i}$$

Since $r_o - r_i = (\sqrt{r_o} + \sqrt{r_i})(\sqrt{r_o} - \sqrt{r_i})$, this reduces to:

$2\sqrt{r_d} = \sqrt{r_o} + \sqrt{r_i}$ QED. Note similarity with

$2 r_c = r_o + r_i$