3 cont'd So, for the LH half:

$$U = 2 \cdot \frac{R}{2EI} \int_0^{3\pi/4} P^2 R^2 (\cos\theta + 1/\sqrt{2})^2 \, d\theta$$

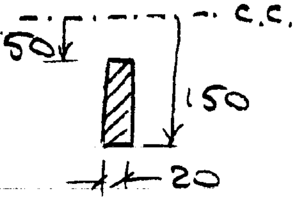
$$= \tfrac{3}{4}(\pi+1) P^2 R^3 / EI$$

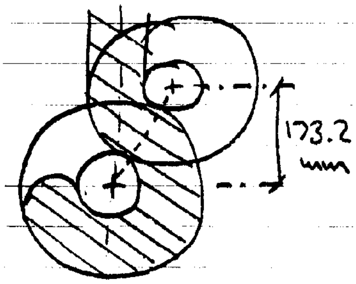
$$\therefore \delta = \frac{\partial U}{\partial P} = \tfrac{3}{2}(\pi+1) P R^3/EI \quad \& \quad = \sqrt{2} R \alpha T \text{ above}$$

$$\text{So } P = \frac{2\sqrt{2}}{3(\pi+1)} EI \alpha T / R^2 = 0.228 \, EI \alpha T/R^2$$

Note that $I \neq \frac{\pi}{64}(D_o^4 - D_i^4)$ because the pipe distorts into ellipse, causing more flexibility – see Burr for more details.

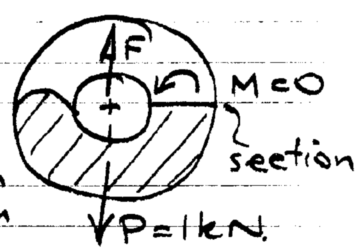
4 Section properties $b = 20$ $r_i = 50$ $r_o = 150$
$A = 2000$ mm^2 $r_c = 100$ mm $r_d = 91.02$ mm





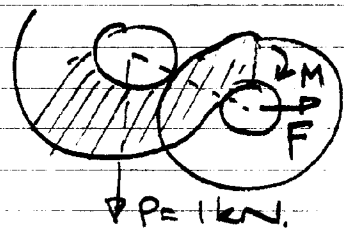
The overall geometry is as shown, the distance between the two centres of curvature being noted. Assume a dummy load of say 1kN & evaluate corresponding stresses. Then scale to 100 MPa.

AXIAL LOAD Stress resultants F & M at section, in conventional senses.



From equilibrium, F = 1kN, M = 0.
From (3): $r_n = 100$ mm $\bar{\sigma} = -5.07$ MPa
From (2) $\sigma = 5.07$ MPa at $r = 50$ mm
& $\sigma = -1.69$ MPa at $r = 150$ mm

TRANSVERSE LOAD. Stress resultants F = 0 (force equilibrium by a irrelevant shear force) and
$M = 1 \times 173.2 = 173.2$ Nm.



From (3) $r_n = 91.02$ mm $\bar{\sigma} = 1.65$ MPa
From (2) $\sigma = -7.92$ MPa at $r = 50$mm
& $\sigma = 3.79$ MPa at $r = 150$ mm

So, for both load locations, $|\hat{\sigma}| = 7.92$ MPa for P = 1kN
$\therefore$ max. load for 100 MPa is $1 \times 100/7.92 =$ 12.6 kN